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IMPACT OF TEMPERATURE VARIATION ON SPATIAL POPULATION DYNAMICS OF *AEDES AEGYPTI*

IMPACTO DA VARIAÇÃO DE TEMPERATURA NA DINÂMICA POPULACIONAL ESPACIAL DO AEDES AEGYPTI

Monalisa Reis da Silva[1]

Pedro Henrique Gasparetto Lugão[2]

Grigori Chapiro [3]

Abstract: This work aims to study a model of partial differential equations (PDE) for the population dynamics of the *Aedes aegypti* mosquito. We propose a numerical resolution using finite volumes. We evaluated the influence of temperature in modeling the parameters and the results for simulations at three different temperatures. The obtained results encourage a discussion about the importance of prevention during the rainy season and compare the cases of dengue during the first thirty epidemiological weeks of two thousand and nineteen.

***Keywords*:** Population dynamics. Partial differential equations. *Aedes aegypti*.

Resumo: Este trabalho tem como objetivo estudar um modelo de equações diferenciais parciais (EDP) para a dinâmica populacional do mosquito *Aedes aegypti*. Propomos uma resolução numérica utilizando volumes finitos. Avaliamos a inflência da temperatura na modelagem dos parâmetros e os resultados para simulações em três diferentes temperaturas. Os resultados obtidos fomentam uma discussão a respeito da importância da prevenção durante a estação chuvosa e apresentam uma comparação com os casos de dengue durante as trinta primeiras semanas epidemiológicas de dois mil e dezenove.

***Palavras-chave*:** Dinâmica populacional. Equações diferenciais parciais. *Aedes aegypti*.

[1]Doutoranda em Modelagem Computacional, Instituto Federal de Educação Ciênica e Tecnologia do Sudeste de Minas Gerais, monalisa.silva@ifsudestemg.edu.br.
[2]Doutorando em Modelagem Computacional, Laboratório Nacional de Computação Científica, pedrolug@lncc.br.
[3]Doutor em Matemática, Universidade Federal de Juiz de Fora, grigori@ice.ufjf.br.

1 INTRODUCTION

The mosquito *Aedes aegypti* (Linnaeus, 1762) is the primary transmitting vector of Dengue, Zika, Chikungunya, and Yellow fever (PAIXÃO; TEIXEIRA; RODRIGUES, 2018). Dengue is considered one of the vector-borne diseases that has spread most rapidly in the world (ORGANIZATION, 2010), causing an estimated 25,000 deaths and an enormous economic cost. In the last 50 years, this endemic disease has grown 30 times, expanding geographically to new countries and, in the current decade, from urban to rural (BESERRA *et al.*, 2014). Urbanization and international travel are key factors that facilitate the spread of these diseases. The study of the spread of mosquitoes and viruses has important implications for the understanding of diseases and hyperendemicity patterns, facilitating the planning of public health actions and vaccine development strategies (MESSINA *et al.*, 2014).

According to the bulletin of the health surveillance secretary (SAÚDE, 2019) until the 30th epidemiological week in 2019, 1,393,062 probable Dengue cases were registered in Brazil. Table 1 shows the division of probable cases until epidemiological week 30, by region of Brazil in 2019, and the average temperature by region of Brazil in 2019 (INMET, 2019).

Table 1: Table of probable cases of dengue and average temperature of 2019.

Region	Probable Cases	Temperature (INMET, 2019)
Northeast	158.792	$30 - 34^o$ C
North	26.134	$30 - 34^o$ C
Midwest	182.809	$28 - 32^0$ C
Southeast	478.499	$22 - 26^o$ C
South	46.828	$16 - 20^o$ C

Temperature is an important ecological factor that influences the insect population establishment, either directly through their development, or indirectly through their feeding (BESERRA *et al.*, 2009). This factor is essential to the development of bioecological studies on the mosquito population dynamics, improving Ae. *aegypti* predictive models, important in areas vulnerable to infestation. In (BESERRA *et al.*, 2009), the effect of temperature on the life cycle of Ae. *aegypti* was evaluated in order to determine thermal requirements for its development and estimate the number of annual generations in the field.

In search of more viable strategies, genetic manipulation was introduced in an attempt to reduce the population of Ae. *aegypti*. The typical strategy consists of releasing genetically modified males carrying a lethal gene, producing the tTAV protein, and causing the death of the descendants, before the adult stage (RUFFATO; CONTE, 2015).

There are several approaches to modeling the population dynamics of Ae. *aegypti*. A mathematical model (YANG *et al.*, 2014) based on ordinary differential equations (ODE) study the importance of temperature and precipitation in mosquito population patterns. In (MCCORMACK; GHANI; FERGUSON, 2019) vectorial transmission of diseases is studied using ordinary differential equations (ODE). Some authors like (YAMASHITA; DAS; CHAPIRO, 2018; YAMASHITA; TAKAHASHI; CHAPIRO, 2018; SILVA; CHAPIRO; PREZOTO, 2020) study the life cycle of Ae. *aegypti* using partial differential equations (PDEs). Such an approach presents advantages when compared to ODEs due to the mosquitoes displacement and by allowing to take into account the terrain's topography. Thus, considering that the erratic movement can be modeled as a diffusion process, a study using PDEs can assist public services in the analysis of actions during epidemics and planning the application of insecticides.

The purpose of this work is to analyze a model with PDEs that studies the population dynamics of Ae. *aegypti* to assess the impact of temperature on mosquito proliferation and compare it with the data provided by monitoring the spread of dengue in Brazil in the first 30 epidemiological weeks of 2019.

2 MODELING AND METHODS

2.1 Modeling

We study the model with two phases in the Ae. *aegypti* life cycle: the immobile and the mobile. We are interested in an urban spatial scale, where diffusion represents the dispersion of mosquitoes due to autonomous and random movements of winged females.

In the Model (1), M and A represent the population density of Ae. *aegypti* mosquitoes in the mobile and immobile phases, respectively. In the mobile phase, only the female mosquito population in the reproductive phase is considered, and in the immobile phase, the egg, larvae, and pupae phases are considered. μ_1 and μ_2 represent the mortality of the mobile and immobile phases, respectively; r represents the female oviposition rate; D is the females' diffusion coefficient; γ is the maturation rate of the immobile phase.

$$\begin{cases} \frac{\partial M}{\partial t} &= \nabla \cdot (D \nabla M) + \gamma A - \mu_1 M, \\ \frac{\partial A}{\partial t} &= rM - (\mu_2 + \gamma) A, \end{cases} \tag{1}$$

with initial conditions give by:

$$M(\cdot, 0) = M_0(\cdot), \quad A(\cdot, 0) = A_0(\cdot). \tag{2}$$

The Model (1) is based on (YAMASHITA; DAS; CHAPIRO, 2018), neglecting the support capacity for the mobile phase and wind velocity. The support capacity term presented in many models was not considered in the mobile phases of the model (1), as there are no limitations for this phase (MCCORMACK; GHANI; FERGUSON, 2019). When the female does the laying, she places the number of eggs that the place holds, if there is no more space, she migrates to other environments to finish laying the eggs, (MCCORMACK; GHANI; FERGUSON, 2019). Thus, it makes sense to consider the space limitation for the development of pupae and larvae (SILVA; LUGÃO; CHAPIRO, 2020). However, as our immobile phase includes larvae, pupae, and eggs, we neglect the capacity term. The lack of the term of carrying capacity for part of the aquatic phase (larvae and pupae) can lead to population explosion, although this model can be used for short times, (SILVA; LUGÃO; CHAPIRO, 2020). The convective term referred to in (YAMASHITA; TAKAHASHI; CHAPIRO, 2018) is related to the wind's velocity. However, it was observed in (OLIVEIRA, 2015) that the wind does not influence the mosquito population growth since by identifying that the wind currents present unfavorable conditions, mosquitoes hide in safe places.

An advantage of this type of modeling is the fact that the topography of the terrain can be taken into account, as well as the influence of temperature on population growth.

2.2 Parameters

To estimate parameter value dependence on temperature we use data for municipality Boqueirão, (BESERRA *et al.*, 2009) for 18^o C, 26^o C and 34^o C. The immobile phase of the model studied here considers egg, larvae, and pupae phases. To obtain the immobile phase's maturation rate, we consider the total duration of the immobile phase as a sum of the average number of days that each of these three phases lasted at each temperature.

Using the number of eggs per female and their average life span at each temperature (BESERRA *et al.*, 2009), we estimate the oviposition rate as the ratio of the number of eggs laid per female to the lifetime obtaining the number of eggs laid per day.

To calculate the mortality rate of the immobile phase, we calculate the mean of the mortality rates of the egg, larvae, and pupae phases (BESERRA *et al.*, 2009). Mortality rates of the female and male phases are calculated as a ratio to the life span at each temperature using data from (BESERRA *et al.*, 2009). The mortality rate of the genetically modified male mosquitoes was considered equal to that of wild male mosquitoes.

The distance traveled by the mosquito varies between 30 and 100 meters when there is availability of breeding places and between 600 and 1000 meters when there is a lack of such places, (OLIVEIRA, 2015). This availability varies directly with the temperature (HONÓRIO *et al.*, 2003). In the dry season, the mosquito travels more, seeking places to lay, and in the rainy season, it travels less due to the abundance of breeding places. For the present modeling we consider that in seven days mosquitoes travel up to 800 m at 18^o C; up to 65 m at 26^o C; and up to 150 m at 34^o C.

So, to calculate the diffusion coefficient, consider the Equation (3) with only the diffusive term, in a single dimension:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}. \tag{3}$$

Considering a local concentration of mosquitoes at $x = 0$ as an initial condition, which can be modeled mathematically as the Dirac delta function, a solution of the heat equation is:

$$u(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right). \tag{4}$$

The function $R(t)$ is the radius approximation of the region reached by mosquitoes at a given time. Since the integral of the Gaussian function in the entire spatial domain is equal to 1 for $t > 0$, we define $R(t)$ as the radius of the region centered in 0 where we find 90% of the initial mosquitoes density. The function $R(t)$ can be calculated using:

$$\int_{-R(t)}^{R(t)} u(x,t)dx = 0.9 \quad \Rightarrow \quad R(t) = erf^{-1}(0.9)\sqrt{4Dt}. \tag{5}$$

The parameter values are summarized in Table 2 and are used in the simulations of the Model (1) presented in Section 2.1.

Table 2: Estimated parameter values for different temperatures.

	18^o C	26^o C	34^o C
r	2.14155 day^{-1}	23.982 day^{-1}	7.262 day^{-1}
γ	0.031046 day^{-1}	0.067249 day^{-1}	0.106269 day^{-1}
D	16900 m^2/day	111 m^2/day	594 m^2/day
μ_1	0.04 day^{-1}	0.04 day^{-1}	0.07 day^{-1}
μ_2	0.22 day^{-1}	0.05 day^{-1}	0.16 day^{-1}

3 NUMERICAL MODELING

The equations describing the population dynamics of Ae. *aegypti* were discretized using the standard finite volume method (FVM), see (LEVEQUE *et al.*, 2002). We consider the domain given by $\Omega = [0, L] \times [0, L]$. The System (1) can be rewritten as:

$$\frac{\partial U(x,y,t)}{\partial t} = \nabla \cdot (\mathbf{D}\nabla U(x,y,t)) + \phi(U,x,y,t), \tag{6}$$

where $U = [M \;\; A]^T$, $\mathbf{D} = [D \quad 0]^T$ (since there is no diffusive term for A) and the remaining terms on the right side of the System (1) are represented as ϕ, the source term.

To solve the problem, we divide the domain Ω into "cells" or control volumes. To formalize the discrete problem we integrate each term of Equation (6) at the cell centered in (x_i, y_j). The integration of the left side results in

$$\int_{y_{j-1/2}}^{y_{j+1/2}} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial U(x,y,t)}{\partial t} dxdy \approx \Delta x_i \Delta y_j \frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta t}, \tag{7}$$

where $U_{i,j}^n = U(x_i, y_j, t_n)$. The integral for the source term is

$$\int_{y_{j-1/2}}^{y_{j+1/2}} \int_{x_{i-1/2}}^{x_{i+1/2}} \phi dxdy \approx \Delta x_i \Delta y_j \phi_{i,j}^n. \tag{8}$$

We only need to consider the term that contains ∇M in Equation (6). The integral of the term in the x direction can be approximated as:

$$\int_{y_{j-1/2}}^{y_{j+1/2}} \int_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial}{\partial x}\left(\frac{\partial (DM)}{\partial x}\right) dxdy = \tag{9}$$

$$D \int_{y_{j-1/2}}^{y_{j+1/2}} \left[\left(\frac{\partial M}{\partial x}\right)_{x_{i+1/2}} - \left(\frac{\partial M}{\partial x}\right)_{x_{i-1/2}}\right] dy \approx \tag{10}$$

$$\Delta y_j D \left[\left(\frac{M_{(x_{i+1},y_j)} - M_{(x_i,y_j)}}{\Delta x_i}\right) - \left(\frac{M_{(x_i,y_j)} - M_{(x_{i-1},y_j)}}{\Delta x_i}\right)\right]. \tag{11}$$

Note that the previous development can be made analogously for the y direction.

Finally, we can isolate M_{ij}^{n+1} and A_{ij}^{n+1} to obtain the solution for each time step given M_{ij}^n and M_{ij}^n from the previous step:

$$\begin{cases} M_{ij}^{n+1} & = M_{ij}^n + \frac{\Delta t D(M_{ij}^n)}{\Delta x_i \Delta y_j} + \Delta t(\gamma A_{ij}^n - \mu_1 M_{ij}^n), \\ A_{ij}^{n+1} & = A_{ij}^n + \Delta t(r M_{ij}^n - \mu_2 A_{ij}^n - \gamma A_{ij}^n). \end{cases} \tag{12}$$

More details on this method can be found in (LEVEQUE *et al.*, 2002; CHAI; LEE; PATANKAR, 1994; PATANKAR, 2018).

4 NUMERICAL RESULTS

We simulate the process of population dispersion and growth for seven days. Numerical simulation considered a region 2000 m $\times$ 2000 m with an initial population of mosquitoes in the winged phase concentrated in the center of the region, see Figure 1 (a). The initial population in the immobile phase is considered zero. For discretization, we use a 40×40 spatial mesh and time step $\Delta t = 0.001$.

We did three different simulations, representing the different temperatures with parameter values as in Table 2. The numerical results after seven days are plotted in figures 1(b), 1(c) and 1(d).

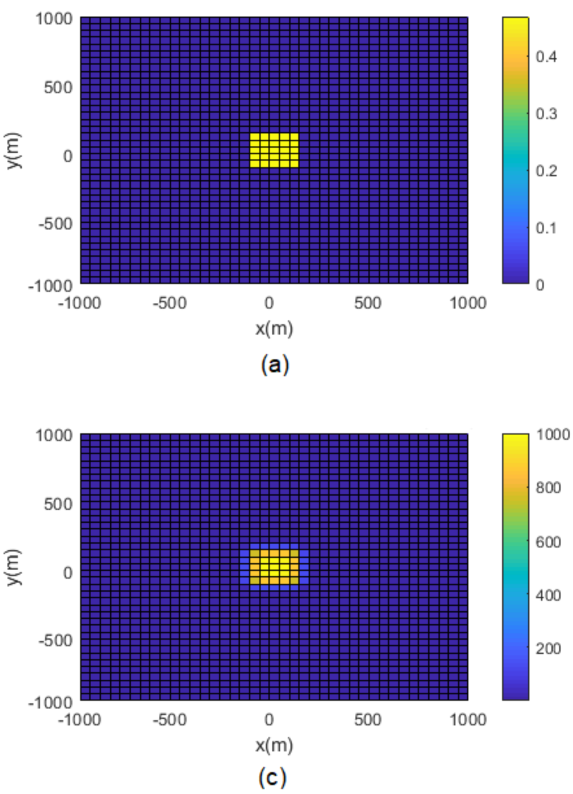

Figure 1: Winged phase population density in 4 different situations: (a) initial state of the simulations, distribution of the population after 7 days simulated with parameters as a function of the temperature of the, (b) 18^o C, (c) 26^o C and (d) 34^o C.

We can observe that during the coldest period, there is a larger dispersion of mosquitoes, and the values of population density are lower than for other temperatures. Integrating the spatial distributions of mosquitoes, we obtain the total population as a function of time. Figure 2 shows that, despite larger dispersion in the coldest period, these temperatures correspond to the lowest population growth. These results agree with data presented in Table 1 for the South region of Brazil, where these temperatures are typical, and where there is the lowest number of Dengue cases.

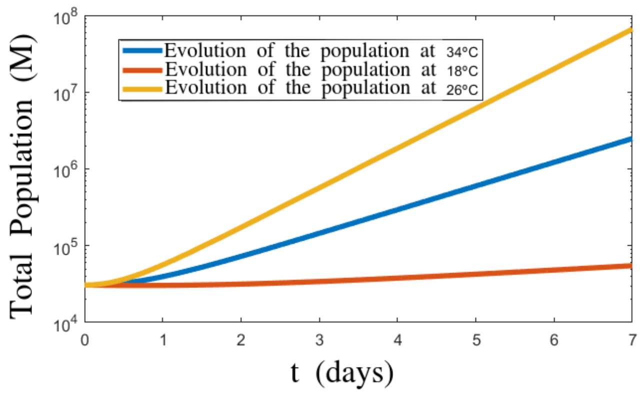

Figure 2: Variation of total female populations over time at different temperatures: 18^o C, 26^o C and 34^o C.

Among the simulated alternatives, the optimum temperature for population growth appears to be 26^o C, which is the maximum average temperature in the Southeast region, where the highest dengue cases are found.

Notice that the periodicity of high and low temperatures throughout the year can play a fundamental role in the spread of vectors since a greater dispersion is observed at low temperatures and greater growth at high temperatures. The successive action of the two phenomena both contributes to the mosquitoes' reaching a larger area of impact and for the total population growth. While during the cold periods, a high incidence of dengue cases may not be perceived, the dispersion of the winged phase increases the range of possible outbreaks in the hot seasons through the distribution of the immobile phase. Such distribution can be seen in the figures 3(a), 3(b) e 3(c).

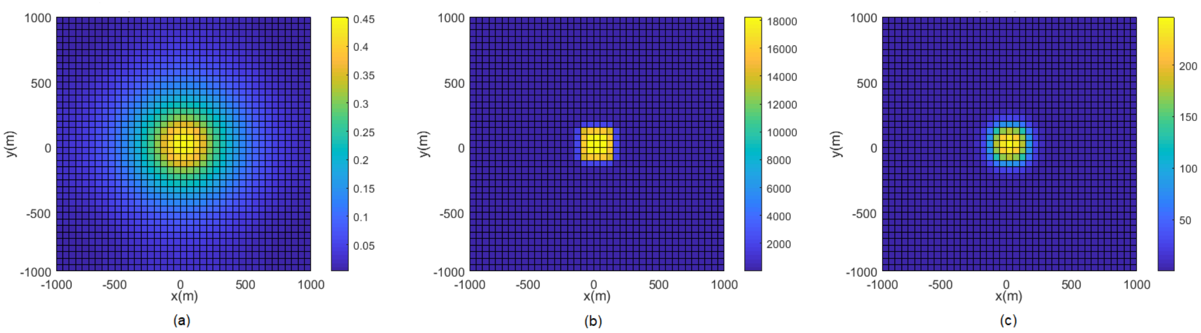

Figure 3: Distribution of the immobile phase after 7 days simulated with the temperature function of the (a) 18^o C, (b) 26^o C and (c) 34^o C.

5 CONCLUSIONS

This study evidences the importance of considering temperature dependence when simulating the population dynamics of Ae. *aegypti*. Using experimental data, we estimated parameter values for the model. Simulations indicate that while high temperatures are associated with population growth, low temperatures season has a big influence in vectors spreading and dispersion. In agreement with the literature, our simulations indicate that the optimal spreading temperatures are around 26^o C, commonly associated with Dengue's highest incidence, as seen in Table 1.

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