MULTIOBJECTIVE PIG IRON COST OPTIMIZATION USING SIMPLEX PROJECTION

OTIMIZAÇÃO MULTIOBJETIVO DO CUSTO DE FERRO GUSA USANDO PROJEÇÃO NO SIMPLEX

Pedro Gabriel da Silva Guimarães[1] iD

Carlos Cristiano Hasenclever Borges[2] iD

Wagner Antonio Arbex[3] iD

Abstract: This work presents a multiobjective optimization model to support the assembly of the raw materials budget for blast furnaces consumption in the production of pig iron, the main material for steelmaking. Given a set of materials and fabrication constraints, such as materials availability, their chemical compositions, the required features for the final product, etc, the objective of the model is to determine the amount of each material that generates the lowest cost solution with minimum wasteful. Due to conflicting objectives, defined by fabrication cost and slag rate, and based on characteristics of decision variables that formulate the problem, some of them in percentage form, an evolutionary multi-objective model has been developed associated with a projection onto a simplex technique aiming to improve the encoding of the genetic solutions. This projection is used in the genetic algorithm-based evolutionary model, wherein each new generated individual, their genes with percentage values are projected onto a simplex, a Euclidean space where the sum of all variables is the unit. The projection allows an intrinsic strategy to deal with percentage constraint of the model variables, increasing significantly the number of feasible individuals generated by the evolutionary procedure. The model has shown to be very effective and useful in determining several scenarios to support decisions making for the raw materials budget.

Keywords: Blast furnace. Hot metal. Multi-objective optimization. Genetic algorithm. Projection onto simplex

[1]Mestre em Ciência da Computação, Universidade Federal de Juiz de Fora, pedro.guimaraes@engenharia.ufjf.br.
[2]Doutor em Engenharia Civil, Universidade Federal de Juiz de Fora, cchborges@ufjf.edu.br.
[3]Doutorado em Engenharia de Sistemas e Computação, Universidade Federal de Juiz de Fora, wagner.arbex@ufjf.edu.br.

Resumo: Este trabalho apresenta um modelo de otimização multiobjetivo para apoiar na construção do orçamento de cargas para o consumo em altos-fornos na fabricação de ferro-gusa, principal material na produção do aço. Dado um conjunto de matérias-primas e restrições de fabricação, como disponibilidade dos materiais, características objetivadas para o produto final, etc, deseja-se calcular a quantidade de cada matéria-prima a ser enfornada que gere as soluções com os menores custos e desperdício. Devido ao interesse em objetivos conflitantes, estes que seriam o custo de fabricação e a taxa de escória produzida, e com base nas características das variáveis, algumas em forma de porcentagens, comum em problemas de misturas, um modelo evolutivo multiobjetivo foi desenvolvido em acoplamento de uma técnica para aprimorar na geração de melhores resultados: a projeção no simplex. Esta projeção é usada no modelo evolutivo baseado em algoritmo genético, onde em cada indivíduo gerado, seus genes com valores em forma de porcentagem são projetados para um simplex, um espaço euclidiano onde a soma de todas as variáveis é igual a 1. Desta forma, elimina-se uma restrição do modelo, aumentando significativamente no número de indivíduos factíveis gerados. O modelo mostrou ser bastante efetivo e útil ao mostrar vários cenários de forma rápida, ajudando na tomada de decisão para a montagem do orçamento de materiais.

Palavras-chave: Alto-forno. gusa. Otimização multiobjetivo. Algoritmo genético. Projeção no simplex.

1   INTRODUCTION

The fabrication of pig iron is a highly complex process and is responsible for the majority of production expenses. The blast furnace is the most important equipment and has the objective to withdraw the oxygen from iron oxides, making a metallic metal with high iron content named pig iron.

With the rising prices, transportation costs, and poor quality of raw materials, many steel companies are implementing low-cost strategies, trying to optimize their production. Several challenges appear in the optimization of this operation, such as logistic problems, supplying chain, burden scheduling, and selection of the raw materials.

The raw materials loaded to blast furnace are the metallic burden (iron ore, pellets, sinter), limestone (bauxite, calcite, dolomite), and fuel (coke or coal). The first provides iron, the second is responsible for removing impurities from the metallic burden, producing the slag, and the last provides heat and energy inside the furnace.

A wide range of materials, with different chemical compositions are available for acquisition, providing many combinations that can satisfy all product requirements and guarantee the stability of the process at affordable prices. However, the selection of a cost-effective arrangement is not an easy task since when purchasing materials, aspects such as availability of the material, their chemical compositions, supplier contract demands, storage condition, etc, should take into account.

This work proposes a multiobjective model to optimize not only the cost of production but also the slag rate, since those are conflicting objectives. The optimization model determines several potential mixtures, allowing a post-decision by the specialist according to the production demands.

2   BLAST FURNACE OPERATION

There are two available procedures for the production of steel products: (*i*) the blast furnace with oxygen steelmaking; (*ii*) based on the electric arc. The former is commonly focused on flat products development, using coke and coal as the main reductant sources, sinter and pellets as the iron-bearing component, while the other procedure uses electric energy to generate the melt scrap. It is important to stand out that the blast furnace steelmaking route is the primary source for worldwide steel production, since the electric arc process route consumes a huge amount of electrical energy, which impacts on production cost (COUDURIER; HOPKINS; WILKOMIRSKY, 1985).

The blast furnace is essentially a continuous counter-current reactor in which the descending materials react with ascending gas derived from the combustion of carbon at the tuyeres. Some iron oxide is involved in direct reaction with carbon, and the gangue and coke ash are fluxed by limestone to form the slag of silica, alumina, and lime. Silica, manganese oxide, and phosphates are reduced, and silicon, manganese, and phosphorus dissolves in the iron. Sulfur passes mainly to the slag as calcium and manganese sulfides, and the iron is saturated with carbon at the temperature in the hearth (GEERDES; TOXOPEUS; VLIET, 2015). Figure 1 shows the main reactions that occur inside a blast furnace.
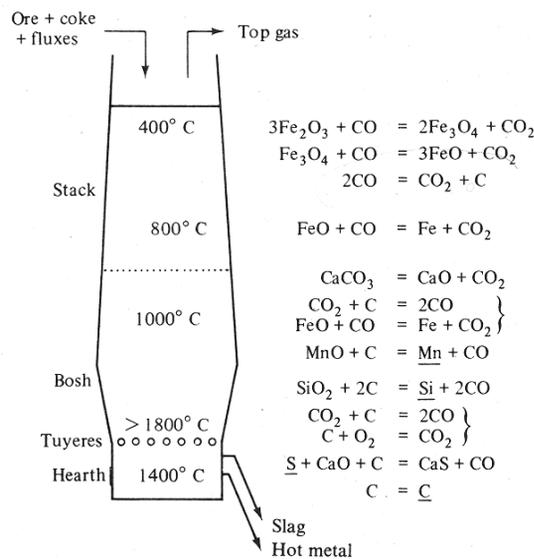

Figure 1: Blast Furnace Reactions.

The blast furnace is not a mass cumulative device, thus all input mass value is equal to the output volume of mass. Accordingly, it is possible to calculate with a certain precision the amount of each material that is necessary to produce the desired products generated by the furnace with the adequate and necessary adjustments. The next section depicted the mathematical model used for blast furnace operation.

3 MATHEMATICAL MODEL

The main objective of the optimization model is to obtain the best BF (blast furnace) burdens with low hot metal cost and low BF slag rate satisfying all the predefined constraints. The deployment of the model is based on material balance. The design of the mathematical optimization model includes the definition of the decision variables, the objective functions to

be optimized, the constraints associated with BF burden, and, finally the developed solving method.

3.1 Decision Variables

The proportions of iron ores and limestone are the decision variables, and the set of feasible solutions can be represented by $V = \{X_1, X_1, .., X_n, Y_1, Y_2, .., Y_m\}$ where X_i is the proportion (%) of i-th iron ore, Y_j is the j-th proportion (%) of limestone, n and m are the number of iron ores and limestones in the burden, respectively.

3.2 Blast Furnace Internal Parameters

Some parameters are needed for calculating the material balance, they are usually acquired from several studies with respect of the blast furnace performance and process control. Those relevant parameters are presented below with their respective values used for the simulations.

(1) C percent content in hot metal (%) - $\%[\text{C}]_{HM} = 4\%$;

(2) Si percent content in hot metal (%) - $\%[\text{Si}]_{HM} = 0.2\%$;

(3) Ratio of Mn that goes to hot metal (%) - $\eta_{\text{Mn}} = 0.2$;

(4) Ratio of Fe that goes to hot metal (%) - $\eta_{\text{Fe}} = 0.997$;

(5) Ratio of amount of iron ore and coal (%) - $\eta_{ore/coal} = 2.72$.

The items (1) and (2) respectively represent the percent content of C and Si that go to the final product, parameters of items (3) and (4) indicate the percent content of Mn and Fe assimilated by the final product, coming from the burden. Item (5) is used to estimate the proportion of coal in the burden.

3.3 Objective Functions

The first objective function determines the cost of production, defined as:

$$MinZ_1 = \sum_{i=1}^{n} C_i A_{ore} \frac{X_i}{100}, \tag{1}$$

where Z_1 represents the cost (unit price) and C_i the price of the i-th raw material (unit price/ton). In this work, only prices for the metallic burden was considered.

The second objective is defined by the slag rate equation:

$$MinZ_2 = \frac{Slag}{HM},\qquad(2)$$

where Z_2 is the slag rate, $Slag$ is the slag volume and HM the amount of hot metal, given by:

$$HM = \frac{\eta_{Fe} \times [Fe] + \eta_{Mn} \times [Mn] + [P]}{1 - \frac{\%[Si]_{HM}}{100} - \frac{\%[C]_{HM}}{100}},\qquad(3)$$

with [Fe], [Mn] and [P] being the total quantities of iron, manganese, and phosphorus in the burden. The quantity of each element can be calculated by the following equations:

$$[E] = \sum_{i=1}^{n} E_i + \sum_{j=1}^{m} E_j + E_{coal} + E_{ICP} - E_{losses},\qquad(4)$$

$$E_i = \frac{\%E_i}{100} A_{ore} \frac{X_i}{100}, \ E_j = \frac{\%E_j}{100} Y_j, \ E_{coal} = \frac{\%E_{coal}}{100} \frac{A_{ore}}{\eta_{ore/coal}}, \ E_{ICP} = \frac{\%E_{ICP}}{100} A_{ICP} \text{ and}$$
$$E_{losses} = \frac{\%E_{losses}}{100} A_{losses},\qquad(5)$$

where E refers to the element E (Al_2O_3, MgO, CaO, Fe, Mn, SiO_2 and P). A_{ICP} and A_{losses} indicate the total proportion of injected coal pulverized and losses, respectively. These two proportion variables are usually set by estimated values.

$$Slag = [SiO_2]_S + [FeO]_S + [MnO]_S + [Al_2O_3] + [CaO] + [MgO],\qquad(6)$$

$$[SiO_2]_S = [SiO_2] - \frac{\%[Si]_{HM}}{100} HM \frac{28 + 32}{28},\qquad(7)$$

$$[FeO]_S = [Fe] \times (1 - \eta_{Fe}) \times \frac{56 + 16}{56},\qquad(8)$$

$$[MnO]_S = [Mn] \times (1 - \eta_{Mn}) \times \frac{55 + 16}{55},\qquad(9)$$

with the variables with subscript S represents the total amount of that element in the BF slag. As the value of amount of hot metal is set, the amount of ore, A_{ore}, may be calculated from the

above equations, as shown below:

$$\#1 = \eta_{\text{Fe}} \frac{\sum_{i=1}^{n} \%\text{Fe} X_i}{10000} + \eta_{\text{Fe}} \frac{\%\text{Fe}_{coal}}{100 \eta_{ore/coal}} \qquad \#4 = \eta_{m}(\sum_{j=1}^{m} \text{Mn}_j - \text{Mn}_{losses})$$

$$\#2 = \frac{\sum_{i=1}^{n} \%\text{P} X_i}{10000} + \frac{\%\text{P}_{coal}}{100 \eta_{ore/coal}} \qquad \#5 = \eta_{\text{Fe}}(\sum_{j=1}^{m} \text{Fe}_j + \text{Fe}_{ICP} - \text{Fe}_{losses})$$

$$\#3 = \eta_{\text{Mn}} \frac{\sum_{i=1}^{n} \%\text{Mn} X_i}{10000} + \eta_{\text{Mn}} \frac{\%\text{Mn}_{coal}}{100 \eta_{ore/coal}} \qquad \#6 = \sum_{j=1}^{m} \text{P}_j + \text{P}_{ICP} - \text{P}_{losses}$$

$$A_{ore} = \frac{HM(1 - \frac{\%\text{Si}_{HM}}{100} - \frac{\%\text{C}_{HM}}{100}) - \#4 - \#5 - \#6}{\#1 + \#2 + \#3} \qquad (10)$$

3.4 Constraints

The optimization of the BF burden respected to the defined objective functions and decision variables should generate feasible solutions in relation to a reasonable types of constraints associated with the used materials and the BF operation. Following, those constraints are depicted.

3.4.1 Ratio Scope of Decision Variables

The ratio scope of each raw material is generally set according to the inventory condition, but also to the situation of the suppliers, for example, a determined kind of material could have a limited amount of purchasing or, by contract, a material must be present with a determined percentage of the burden in a pre-defined period. Thus, some variables could be limited.

$$LL_i \leq V_i \leq UL_i, \qquad (11)$$

where: LL_i and UL_i represent the lower and upper limit ratio of the i-th decision variable.

3.4.2 Iron Ore Proportions

As the variables associated to the iron ore are in percentage, the total sum must respect the quantity of 100%, resulting in a equality constraint that should be enforced:

$$\sum_{i=1}^{n} X_i = 100\% \qquad (12)$$

3.4.3 Slag Basicity

The slag basicity represents the ratio between basic oxides and acid of the slag, in this case, in the binary form (CaO and SiO$_2$). This value must respect a reference value which indicates that the slag has efficient desulfurization and will come out easily from the furnace.

$$\frac{[\text{CaO}]}{[\text{SiO}_2]_S} = {}^{\text{CaO}}\!/_{\text{SiO}_2}. \tag{13}$$

Here, it is important to highlight that the right side of the constraint equation is a pre-defined reference value, as the left side is determined by the model variables.

3.4.4 Slag Volume

The amount of slag depends on the quality of raw materials. A better quality of materials provides a lower slag amount with a more stable furnace. This constraint limits the amount of slag in kg.

$$Slag_{Min} \leq Slag \leq Slag_{Max}, \tag{14}$$

where $Slag_{Max}$ and $Slag_{Min}$ is the maximum and minimum of BF slag.

3.4.5 Element Percent Content in the Slag

The elements that form the slag, as presented in Equation (6), each one of them may be limited in the slag to control its viscosity or basicity.

$$\%[\text{E}]_{S_{LL}} \leq \frac{[\text{E}]_S}{Slag} \times 100\% \leq \%[\text{E}]_{S_{UL}}, \tag{15}$$

with $\%[\text{E}]_{S_{LL}}$ and $\%[\text{E}]_{S_{UL}}$ representing the lower and upper limit ratio of the element in the BF slag (%).

4 SOLUTION USING AN EVOLUTIONARY MULTIOBJECTIVE ALGORITHM

This work uses the well-know NSGA-2 algorithm, presented by (DEB *et al.*, 2002), which is an algorithm for solving multiobjective optimization problems. It is based on a generational genetic algorithm and constructed using two important mechanisms on the selection process: non-nominated sorting and crowding distance (see Fig. 2).

Non-dominated sorting classifies the population in levels according to the number of

domination of the individuals in relation to the whole population. An individual is defined as non-dominated when no other individual in the population is better evaluated in relation to the objective functions of the problem. The NSGA-2 uses the level of dominance of the individuals for the ranking process, determining the non-dominated individuals of the population and defining a dummy fitness for these individuals which are removed from the population.. Once again, the new non-dominated individuals are determined, receiving a worst dummy fitness in relation to the previously non-dominated ranked individuals This process is performed until all individuals in the population are ranked.

After all individuals in the population are ranked, the crowding distance of each individual is calculated, determined by the sum of Euclidean distances from individuals of the same level of dominance. Thus, at each dominance/rank level, individuals are sorted by crowding distance, which implies that individuals in regions of the search space with lower population density, will be more likely to be selected, allowing the algorithm to better distribute the solutions along the search space, preventing concentration of solutions in specific regions of the Pareto front (DEB *et al.*, 2002). Therefore, individuals with lower dominance/rank values and higher crowding distance have more probability to be selected for the next generations.
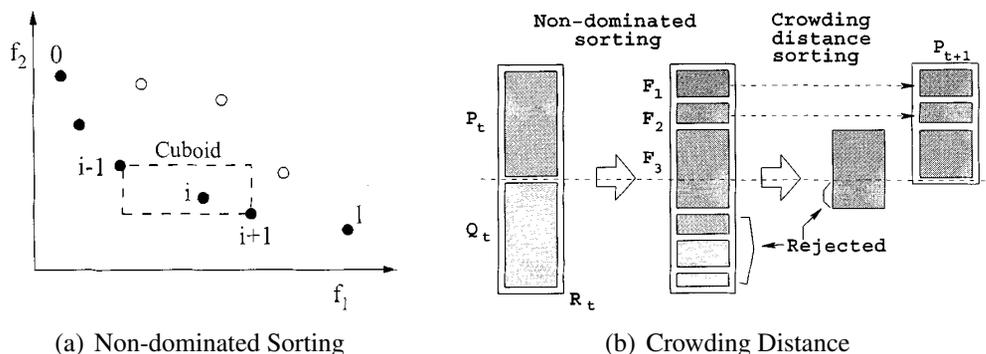

(a) Non-dominated Sorting   (b) Crowding Distance

Figure 2: NSGA-2.

4.1 Initial Population

The chromosome solution is represented by V. For one chromosome, each gene is produced as followed:

$$V_{1,...,k} = U(UL_{1,...,k} \sim LL_{1,...,k}), \qquad (16)$$

where $U(UL_{1,...,k} \sim LL_{1,...,k})$ produces a uniform distribution random number between $LL_{1,...,k}$ and $UL_{1,...,k}$, lower and upper limit for the genes $1, \ldots, k$.

4.2 Genetic Operators

To improve the generation of good individuals in the NSGA-2, some adjustments on genetic operators were necessary.

Firstly, the selection of the individuals, the model adopts the tournament procedure,i.e.: pick randomly two chromosomes, compare the ranks of them and the lower rank wins the tournament. If the chosen individuals have the same rank, the chromosome with a higher crowding distance is selected. Hence, a chromosome less dominated and located in a less density region has more probability to be selected and, consequently, to reproduce and mutate, generating better solutions.

For the mutation of a selected chromosome, the following strategy is used for all its genes:

$$\sigma_{1,\dots,k} = \gamma \times (UL_{1,\dots,k} - LL_{1,\dots,k}) \text{ if } dir = 1, \tag{17}$$

$$y_{1,\dots,k} \sim U(V_{1,\dots,k}, \sigma_{1,\dots,k}) \text{ if } dir = 1, \tag{18}$$

$$\sigma_{kmut} = \gamma_{kmut} \times (UL_{kmut} - LL_{kmut}) \text{ if } dir = 0, \tag{19}$$

$$y_{kmut} \sim U(V_{kmut}, \sigma_{kmut}) \text{ if } dir = 0, \tag{20}$$

where σ are the variances adopted to generate random numbers with uniform distribution, generating a new chromosome y. The γ parameter limits the size of σ.The dir binary random variable decides whether all genes, 1 to k, will suffer mutation or only the gene $kmut$, randomly selected. For the experiments, the mutation rate is set to 25% and γ to 0.05.

For the crossover, two chromosomes are also picked from the tournament selection. The following strategy is applied to all genes:

$$\alpha \sim U(0, \sigma), \tag{21}$$

$$y_{1,\dots,k}^i = \alpha \times V_{1,\dots,k}^i (1 - \alpha) \times V_{1,\dots,k}^j, \tag{22}$$

$$y_{1,\dots,k}^j = \alpha \times V_{1,\dots,k}^j (1 - \alpha) \times V_{1,\dots,k}^i, \tag{23}$$

where the parameter σ is the variance to the linear coefficient generated by random number with uniform distribution, given by α. The values V^i and V^j are the picked chromosomes for the crossover and y^i and y^j are their offspring. For the experiments, the crossover rate was set 75% and the σ value is 0.5.

4.3 Penalized Objective Functions

The penalized objective functions used in the multiobjective optimization are composed by the previous defined objective functions coupled with the equality and inequality constraints in a standard mode:

$$f_j(x) = \min Z_j + k \sum_i^I \max(0, g_i(x))^2 + k \sum_i^J \max(0, |h_i(x)| - \delta_i)^2, \qquad (24)$$

where $f_j(x)$ is the j-th fitness function. The function $g_i(x)$ represents the i-th inequality constraint using variable x and the function $h_i(x)$ represents the i-th equality constraint with x. The parameter k is the coefficient of penalization, set as a constant value of 10^{10}. The constant δ_i is the tolerance margin for the i-th equality constraint when violated. The two equality constraints, represented by Equation (13) and Equation (12), receive the values $\delta_1 = 10^{-3}$ and $\delta_2 = 10^{-4}$, respectively.

5 HANDLING PERCENTAGES VARIABLES (PROJECTION ONTO A SIMPLEX)

The decision variables related to iron ore are defined as percentages (%), with their sum must be 100%. This situation is commonly present in blending optimization problems, where the objective usually is to find the lowest mixture of materials cost.

Those kinds of problems in steelmaking are more used in sintering process, as seen in (CAO *et al.*, 2013), but also in cement, food, chemical, pharmaceutical, oil industries problems. For the most part, these are linear problems, which are easily solved by linear programming methods.

According to Lu *et al.* (2007), in the sintering process, which is very similar to the problem of this work, with the increasing complexity of the problem solving, the linear programming method lose efficiency, and even can not be able to solve the problem correctly. The use of meta-heuristics methods as genetic algorithm, particle swarm, bee colony, etc, have potential to provide multiobjective solutions and could be more practical than LP methods.

As defined in (SALMERON *et al.*, 2016), the set of possible mixtures may be mathematically defined by a unit simplex, such as:

$$\Delta^n := \left\{ x = (x_1, ... x_n)^T \in \mathbb{R}^n : 0 \leq x_i \leq 1, i = 1, ..., n, \text{and} \sum_{i=1}^n x_i = 1 \right\} \qquad (25)$$

where the variables x_i represent the i-th fraction of the component to compose the entire variable x. Figure 3 shows a graphical example of the search space in 2D (left hand side) and 3D (right hand side) consisting of unit simplices removing the space where the minimum dose constraint is not satisfied.
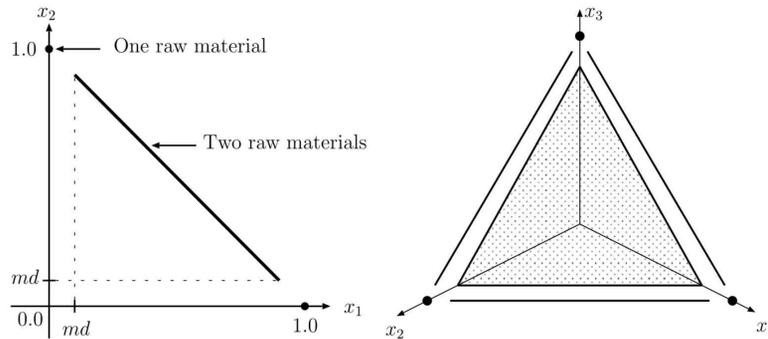

Figure 3: 2D and 3D simplices removing the minimum dose region.

At first, it is possible to use the Equation (12) to keep the sum of x. However, if the population initialization or the mutation and crossover operators generate a large number of individuals that violate the percentage constraint, the convergence speed will be affected considerably. The most usual solution for that is to normalize all chromosomes, as adopted in (TOKLU, 2005), with all genes multiplied by their respective normalization factors.

Nonetheless, when x has bounds constraints as shown in Equation (11), a normalization sometimes can generate chromosomes that respect those bounds while keeping the sum, as consequence, many of the individuals are practically discarded from the next generation. This paper proposes a projection scheme of the solution with bounds constraints onto a simplex.

The projection of a point y onto the simplex Δ^n is obtained by solving the minimization problem:

$$x = arg \min_{x \in \Delta^n} ||x - y||. \tag{26}$$

The algorithm presented here is an adaptation of (CHEN; YE, 2011), where some adjustments to work with bounds constraints variables are implemented providing a significant impact on the convergence speed, with few iterations necessary to generate non-dominated solutions in relation to the size of the population.

The algorithm with adjustments is presented below:

Algorithm 1: Projection onto simplex with box constraints.

1 Function PROJSPLX-C (y,y_{min},y_{max},δ):

    Input: $y = (y_1, \ldots, y_n)^T \in \mathbb{R}^n, y_{min}, y_{max}, \delta$

    Output: x

3 $x \leftarrow y$;

4   **repeat**

6 $y' \leftarrow x$;

8 $x \leftarrow$ PROJSPLX (y') ;

10 $x \leftarrow \min(\max(x, y_{min}), y_{max})$;

11   **until** $|(\sum_{i=1}^{n} x_i) - 1| < \delta$;

13   **return** x;

Basically, the algorithm executes the projection proposed in (CHEN; YE, 2011) (PROJSPLX), applying, after that, the bound constraints of the obtained projected solution until the difference of the sum and the unit is lower than tolerance δ. The parameter δ represents the required precision (in the experiments, $\delta = 1 \times 10^{-10}$) and y_{min} e y_{max} refers to both decision variables lower and upper limit.

6 NUMERICAL EXPERIMENTS

The optimization model was coded in C# .NET and based on actual raw materials compositions. The obtained results are presented in this section.

6.1 Starting Conditions

The raw materials chosen for this model are shown in Table 1. There are five kinds of iron ores (M1, M2, M3, M4, M5) and three limestones (Bauxite, Dolomite, and Calcite). The raw materials proportions are the decision variables that the optimization will manipulate. The ratio scope of the variables are also presented in Table 1. Scrap and PCI have their proportions set with constant values in this experiment. The product requirements which are the constraints of the model are shown in Table 3. The flowchart of optimization model is shown below.
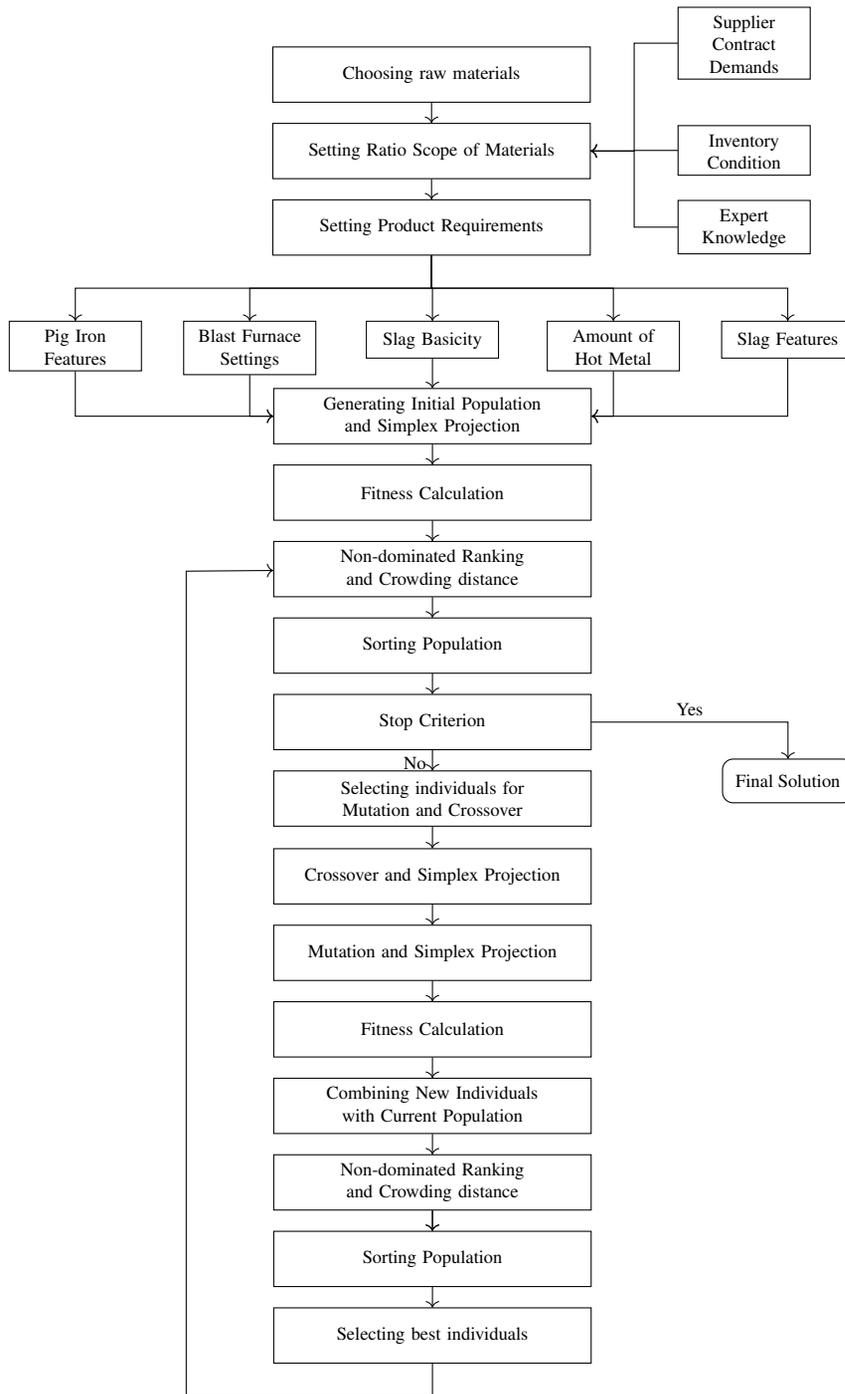
Figure 4: Flow chart of the optimization model.

Table 1: Conditions of raw materials.

Project	Proportion (% or kg)	Price (unit price/t)	Fe (%)	SiO$_2$ (%)	Al$_2$O$_3$ (%)	P (%)	Mn (%)	CaO (%)	MgO (%)	C (%)	Min	Max
M1		108.57	58.20	13.60	1.67	0.07	0.01				0.00%	25.00%
M2		154.33	59.00	10.50	1.50	0.10	0.09				0.00%	50.00%
M3		173.81	60.50	6.50	2.25	0.09	0.06				0.00%	10.00%
M4		233.83	63.50	3.00	2.5	0.02	0.49				0.00%	10.00%
M5		324.24	64.45	4.28	0.38	0.03	0.08	2.3	0.51		0.00%	100.00%
Bauxite			6.2	11	37						0 kg	100 kg
Dolomite				2.96	0.86			30.71	16.65		0 kg	100 kg
Calcite				2.8	0.49			52.66	0.37		0 kg	100 kg
Scrap	60		59.42	10.19	2.56			11.98	3.85			
Coal			0.14	0.75	0.15	0.09		1.18	0.15	74.00		
PCI	88		0.56	3.00	0.60	0.35		4.70	0.60	66.00		

Table 2: Conditions of losses.

Project	Proportion (kg)	Fe (%)	SiO$_2$ (%)	Al$_2$O$_3$ (%)	P (%)	Mn (%)	CaO (%)	MgO (%)	C (%)
Dust+Mud	60	55.62	3.66	4.95	0.034	0.2	4.95	0.22	26.54

Table 3: Product Requirements.

Slag Basicity	Slag (kg)	%[SiO$_2$]$_S$ (%)	%[Al$_2$O$_3$]$_S$ (%)	%[MgO]$_S$ (%)	Coal Ratio	Hot Metal (kg)
0.67	145 - 285	31 - 80	12.5 - 17.0	7 - 8	2.72	1000

6.2 Feasibility Rate

With the starting conditions presented, the program was executed two times, one without and another with the projection. The projection is performed every time the proportions variables are modified, on initializing the population, on crossing and mutation. Set for run until 100000 iterations with 100 individuals, the following parameter was used for measuring the performance of the algorithms:

$$M = \frac{1}{n} \sum_{i=1}^{n} \frac{\#pop'_m(i) + \#pop'_c(i)}{\#pop_m(i) + \#pop_c(i)}, \tag{27}$$

where n is the number of iterations, $\#pop'_m$ e $\#pop'_c$ are the number of feasible individuals generated by mutation and crossover, respectively, and $\#pop_m$ e $\#pop_c$ are the total number of individuals generated by the same operators, being feasible or not.

This parameter was called the feasibility rate. Its purpose is to indicate how the implemented operators are behaving, if they are generating a good number of possible candidates or if the convergence is near.

As can be seen in Fig. 5, the algorithm with projection generates feasible individuals much earlier and it has a peak rate greater than the one with no projection.
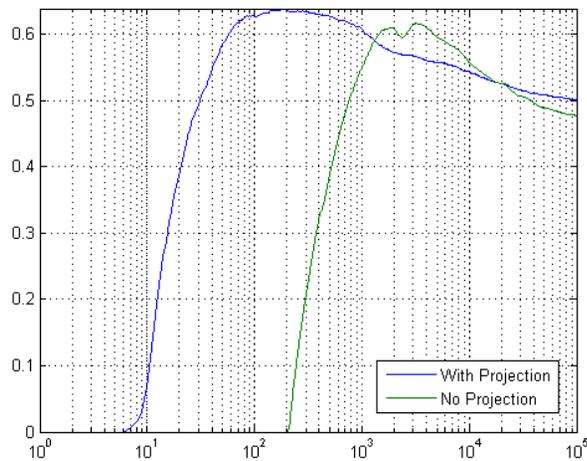

Figure 5: Feasibility Rate.

6.3 Results

As similar as before, both algorithms, one with and one without projection, were executed, the first until 100000 iterations, the second 20000. The results are shown in Fig. 6.
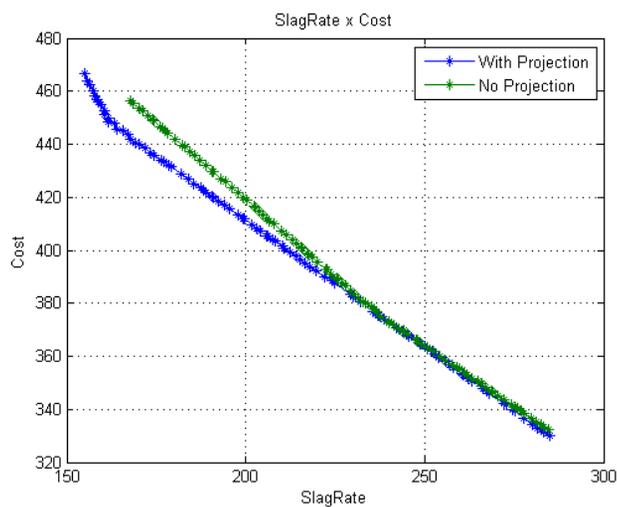

Figure 6: Results of the algorithms.

From the figure above, the algorithm with no projection acquires worse solutions and only between 170 and 285 of Slag Rate, even after 100000 iterations. The algorithm with projection secures better solutions and earlier than the first one. The plots of the variables for the algorithm with projection are shown in the Fig. 7.
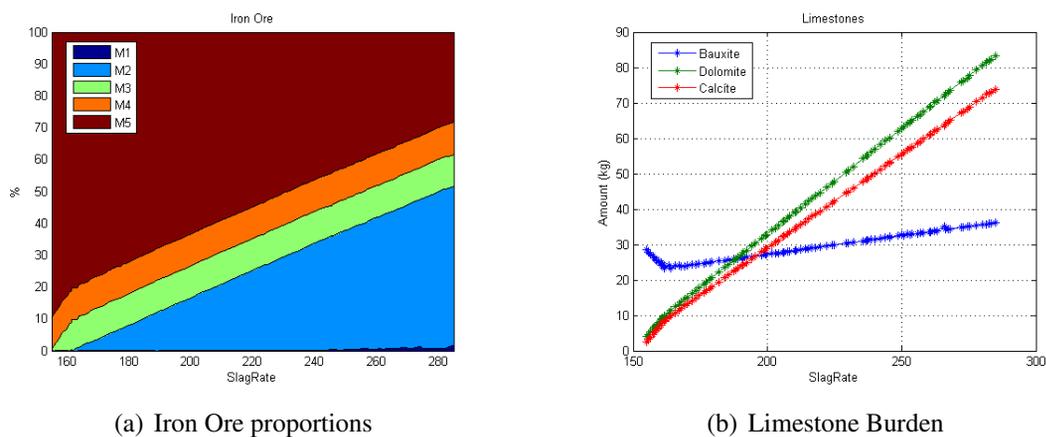

(a) Iron Ore proportions

(b) Limestone Burden

Figure 7: Decision Variables - Projection.

7   CONCLUSIONS

A multi-objective optimization model was developed in this paper and presented an effective way to reduce hot metal cost with efficiency while satisfying all product requirements. The application permits an expert to make a decision under several options in a short time, having a glance at the potential of the raw materials at disposal, allowing him to know which material has more impact on slag rate and price for better raw materials buying analysis.

A stop criterion for the algorithm is still needed to be studied, the current program has an online plot of the non-dominated individuals, so visually, it is possible to see if the solution has stabilized and then the user can stop it if he thinks the results are satisfactory.

Nonetheless, using simplex projection for percentage variables was very effective, however, a close study must be done to see if it is more beneficial than using normalization on blending optimization problems.

REFERENCES

CAO, W. *et al.* A genetic algorithm application to minimize pig iron cost. *ISIJ International*, v. 53, n. 2, p. 207–212, 2013.

CHEN, Y.; YE, X. Projection onto a simplex. *ArXiv*, 2011.

Chapter 7 - iron and steelmaking. In: COUDURIER, L.; HOPKINS, D. W.; WILKOMIRSKY, I. (Ed.). *Fundamentals of Metallurgical Processes*. Second edition. [S.l.]: Pergamon, 1985. p. 246 – 300.

DEB, K. *et al.* A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, v. 6, n. 2, p. 182–197, 2002.

GEERDES, M.; TOXOPEUS, H.; VLIET, C. V. D. Modern blast furnace ironmaking : an introduction. In: *Modern blast furnace ironmaking : an introduction*. Second edition. [S.l.]: IOS Press, 2015.

LU, X. W. *et al.* Research on sintering burdening optimization based on genetic algorithm. v. 42, 2007.

SALMERON, J. G. *et al.* On regular simplex division in solving blending problems. In: *XIII Global Optimization Workshop*. [S.l.: s.n.], 2016.

TOKLU, Y. C. Aggregate blending using genetic algorithms. *Computer–Aided Civil and Infrastructure Engineering*, v. 20, p. 450 – 460, 2005.