ANALYSIS THROUGH CONSTRUCTAL DESIGN OF THE INFLUENCE OF SPACING BETWEEN STIFFENERS IN THE DEFLECTION OF PLATES

ANÁLISE VIA DESIGN CONSTRUTAL DA INFLUÊNCIA DO ESPAÇAMENTO ENTRE ENRIJECEDORES NA DEFLEXÃO DE PLACAS

Milton Cesar Bastos Portela Junior1  
Vinícius Torres Pinto2  
Luiz Alberto Oliveira Rocha3  
Elizaldo Domingues dos Santos4  
Liércio André Isoldi5

Abstract: Using the Finite Element Method (FEM), stiffened plates arrangements defined by the application of the Constructal Design Method (CDM) were analyzed. So that, through an Exhaustive Search (ES), different spacing between the stiffeners were tested regarding the central and maximum deflections. Starting from a non-stiffened plate with a fixed volume, a portion of its material was completely removed from its thickness and transformed into stiffeners considering the volumetric fraction \( \phi = 0.5 \). It were established 4 arrangements: P(2,2), P(2,3), P(3,2) and P(3,3), varying for each one, the spacing between the stiffeners, as well as the parameter \( h_s/t_s \) (ratio between height and thickness of stiffeners). The results showed that stiffeners equally spaced in the longitudinal and transverse directions with higher ratios \( h_s/t_s \) are more effective, being able to reduce the central and maximum deflections by more than 95% compared to the non-stiffened reference plate.

Keywords: Stiffened Plates. Numerical Simulation. Constructal Design. Deflection on plates.

Resumo: Utilizando o Método dos Elementos Finitos (MEF) foram analisados arranjos de placas enrijecidas definidos pela aplicação do Método Design Construtal (MDC). De modo que, através de uma Busca Exaustiva (BE), diferentes espaçamentos entre os enrijecedores foram testados quanto às deflexões centrais e máximas. Partindo de uma placa não enrijecida com volume fixo, uma parcela de seu material foi integralmente removida de sua espessura e transformada em enrijecedores considerando a fração volumétrica \( \phi = 0.5 \). Foram estabelecidos 4 arranjos: P(2,2), P(2,3), P(3,2) e P(3,3), variando para cada um, os espaçamentos entre os enrijecedores, assim como o parâmetro \( h_s/t_s \) (razão entre a altura e a espessura dos enrijecedores). Os resultados mostraram que enrijecedores igualmente espaçados nas direções longitudinal e transversal com maiores razões \( h_s/t_s \) são mais eficazes, podendo reduzir as deflexões centrais e máximas em mais de 95% em comparação com a placa de referência não enrijecida.


1 Mestre em Engenharia Oceânica, Universidade Federal do Rio Grande, portela.milton@gmail.com  
2 Mestre em Engenharia Oceânica, Universidade Federal do Rio Grande, viniciustorreseng@gmail.com  
3 Doutor em Engenharia Mecânica, Universidade do Vale do Rio dos Sinos, luiz@unisinos.br  
4 Doutor em Engenharia Mecânica, Universidade Federal do Rio Grande, elizaldosantos@furg.br  
5 Doutor em Engenharia Mecânica, Universidade Federal do Rio Grande, liercioisoldi@furg.br

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1 INTRODUCTION

Plates are structural components that have a much smaller dimension than the others, i.e., their thickness is very small when compared to the width and length (SZILARD, 2004). Usually the plates work under flexion and/or compression parallel to its plane, being widely used in engineering, such as aerospace, civil, mechanical and naval (BIRMAN, 2011). However, due to its high slenderness it is necessary to insert stiffeners, traditionally fixed in longitudinal and/or transverse directions, whose main function is to increase stiffness.

Analytical solutions for analyzing the mechanical behavior of plates are uncommon, and when exist, usually lead to inaccurate results. Hence, it is interesting to apply numerical methods to solve this type of problem. Rossow and Ibrahimkhail (1978) presented models of stiffened plates with concentric and eccentric stiffeners using the Method of Constraints. Mukhopadhyay and Satsangi (1984) proposed a model based on the Finite Element Method (FEM) for the analysis of stiffened plates employing an isoparametric type finite element. Tanaka and Bercin (1998) used the Boundary Element Method (BEM) to study stiffened plates under bending, considering the effects of bending, torsion, deformation and eccentricity of the stiffeners in the equation. Salomon (2000) explains that depending on the design requirements, the stiffeners can occupy any position on the plate, aiming an effective structural arrangement, which is capable of resist the imposed load, limiting the stresses and deformations to their allowed values.

More recently, Fernandes (2009) presented a numerical model based on the hypotheses of Kirchhoff's theory and applying the BEM to analyze the bending in plates reinforced by beams considering different materials. Benai and Pedatzu (2010) based on orthotropic plate work of Shade (1940, 1941, 1951) implemented a computer program to analyze stiffened plates under bending. Damanpack et al. (2013) developed the analysis of functionally graded stiffened plates through BEM, i.e., plates with material properties variable along the thickness.
In the context of search for better structural performance by means the Constructal Design Method (CDM) associate with FEM, recent studies about steel plates have shown interesting results. Among them, one can highlight the investigations concerning the buckling of perforated plates (HELBIG et al., 2016; and DA SILVA et al., 2019); and stiffened plates (LIMA et al., 2018 and LIMA et al., 2020). Specifically, on stiffened plates under bending, it is possible to mention: De Queiroz et al. (2019), Pinto et al. (2019); and Troina et al. (2020). Finally, the application of the CDM in the structural analysis of aircraft also stands out (MARDANPOUR et al., 2019 and IZADPANAHI et al., 2020).

Therefore, the present study analyzed different arrangements of stiffened plates defined by the application of the CDM, numerically solved by applying the FEM using the ANSYS® software, and compared among each other through the Exhaustive Search (ES) technique. So, considering the variation of spacing between the stiffeners and variation of \( h_s/t_s \) (ratio between height and thickness of stiffeners), it was possible to identify the geometric configuration that minimizes the central and maximum deflections.

2 MATERIAL AND METHODS

2.1 Computational Modeling

The FEM is a numerical method widely applied in the analysis of engineering problems, especially when it is not possible to obtain a precise solution through analytical solutions (COOK et al., 2001; SEGERLIND, 1984). According to Szilard (2004), the FEM consists of the decomposition of the continuous domain into sub-domains of finite size connected by nodal points.

The displacement field of each element is arbitraged according to the displacement of the nodes, replacing the continuous mathematical model by the equilibrium of each finite element, converting the differential equations into algebraic equations in each element. Thus, this global system allows the determination of the approximate solution after the introduction of the loads and boundary conditions (ASSAN, 2003; SORIANO, 2003).
In this study, the FEM was used to solve the proposed plate models by means the ANSYS® software. The SHELL281 finite element was adopted, since it is suitable for modeling thin and moderately thick plates. This two-dimensional element is based on the Reissner-Mindlin hypotheses, having 8 nodes in its quadrilateral version and 6 nodes in its triangular version, with 6 degrees of freedom per node, being 3 rotations and 3 translations in relation to the x, y and z axes (ANSYS, 2019).

2.1.1 Verification and Validation of the Computational Model

For the verification, it was used the case presented in Fig. 1, also solved in Troina et al. (2018) through ANSYS® software using the three-dimensional element SOLID95. The plate was subjected to a uniform transverse loading of 68.95 kPa with boundary conditions of simply supported edges. The material has elastic modulus $E = 206.8427$ GPa and Poisson's ratio $v = 0.3$.

Figure 1 - Rectangular plate with 2 orthogonal stiffeners

The solution was performed using the SHELL281 element in the quadrilateral version with a mesh of 15,200 finite elements, defined after the mesh convergence test shown in Fig. 2, along with the result obtained by Troina et al. (2018) for the central deflection of the plate $U_z$. 

From Fig. 2, a difference of 1.07% was reached between the obtained result of $U_z = 0.281$ mm and the result presented by Troina et al. (2018) of $U_z = 0.278$ mm, verifying proposed the computational model.

In turn, the physical model used for validation is shown in Fig. 3, being developed by Carrijo et al. (2009). The experiment considered a square plate with its four corners simply supported, subjected to a transverse load uniformly distributed of 0.96 kPa. The material has elastic modulus $E = 2.5$ GPa and Poisson’s ratio $\nu = 0.36$.

Employing the SHELL281 quadrilateral element, the experiment was numerically simulated with a mesh that totaled 9,150 finite elements, established after the mesh convergence test presented in Fig. 4, where the result obtained by Carrijo et al. (1999) is also plotted for the central deflection of the plate $U_z$. 
The results of Fig. 4 allow to infer a difference of 4.45% between the numerical model of the present study \( (U_z = 6.51 \text{ mm}) \) and the experimental model \( (U_z = 6.22 \text{ mm}) \), validating the computational modeling.

**Figure 4 - Validation of the computational model**

![Graph showing validation of computational model](image)

### 2.2 Constructal Design Method (CDM)

The Constructal Law is the physical phenomenon behind the vast geometric complexity of the flow systems that exist in nature. These systems generate shape and structure over time in order to facilitate flow access that passes through them. Thus, the geometric design of the system is not a random result; it arises in a natural evolutionary attempt to achieve its best performance (BEJAN AND ZANE, 2008; BEJAN AND LORENTE, 2008).

Based on a principle of restrictions and objectives, the CDM is the practical application of the Construct Law. The performance of a system carries inherent restrictions, which may be the space allocated for its evolution, the material available, as well as limit rates of pressure, temperature and stress. When the problem restrictions are defined, the degrees of freedom related to the geometric parameters are modified, searching an arrangement that achieves the better possible performance according to a predetermined performance indicator (REIS, 2006; DOS SANTOS et al. 2017).

It is possible to find several studies applying the CDM on problems of fluid mechanics and heat transfer. However, its use in structural analysis is still few
explored, having its viability proven in a similar way to applications in fluid mechanics and heat transfer in Bejan e Lorente (2008), Lorente et al. (2010) and Isoldi et al. (2013).

The application of the CDM in current study started from a non-stiffened steel plate taken as reference, with length $a = 2000 \text{ mm}$, width $b = 1000 \text{ mm}$ and thickness $t = 20 \text{ mm}$. Keeping constant the dimensions $a$ and $b$, as well as the total volume of steel, a portion of material deducted entirely from the thickness of the reference plate was transformed into stiffeners using the volumetric fraction $\phi$, mathematically defined by:

$$\phi = \frac{V_s}{V_r} = \frac{N_{ls}(ah_{st}s) + N_{ls}[(b - N_{ls}t_s)h_{st}s]}{abt}$$

(1)

where $V_s$ is the material volume of the stiffeners and $V_r$ is the material volume of the reference plate. The height and thickness of the stiffeners are $h_s$ and $t_s$, respectively; $N_{ls}$ and $N_{ts}$ represent the number of stiffeners in the longitudinal and transverse directions, respectively. Besides that, $a$, $b$ and $t$ are the length, width and thickness of the reference plate, respectively. These parameters are shown in Fig. 5.

Figure 5 - Plate P(2,2)
Adopting a value of $\phi = 0.5$, that is, 50% of the reference plate material transformed into stiffeners, and following the plate identification system $P(N_{ls}, N_{ts})$, 4 stiffened plate arrangements were established: $P(2,2)$, $P(2,3)$, $P(3,2)$ and $P(3,3)$. In each arrangement, the longitudinal spacing $S_{l}$ and transverse spacing $S_{t}$ stiffeners were varied; and for each new adopted spacing the ratio between the height and thickness of the stiffeners $h_{s}/t_{s}$ was also varied, considering different values of commercial steel plates for the stiffeners' thicknesses. Aiming to minimize the central and maximum deflections assumed as the performance indicator of the study. The Fig. 6 shows a schematic of the search space definition by the application of the CDM.

Following the criteria established in Troina et al. (2020), the height of the stiffeners was limited to 300 mm to avoid excessive disproportion in relation to the dimensions of the plate. Likewise, ratios $h_{s}/t_{s}$ less than 1 have been disregarded, avoiding stiffeners with thicknesses greater than heights.

Finally, it is important to mention that all the analyzed plates were subjected to a uniform transverse load of 10 kPa, ensuring an elastic linear behavior. The material was the structural steel A-36 with an elastic modulus of 200 GPa and a Poisson's ratio of 0.30, which, as Mandal (2017) explains, has a relatively low cost, having satisfactory mechanical properties, facilitating the manufacturing and welding processes.
Figure 6 - Application Scheme of the MDC in the case study

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3 RESULTS

To determine the appropriate size of the finite elements used in the discretization of the numerical models analyzed in this study, a mesh convergence test was previously performed, using the plate P(2,2) with $S_l = 222.22$ mm, $S_t = 111.11$ and $h_s/t_s = 20.84$. The size of the finite elements was reduced consecutively with reference to the plate width $b = 1000$ mm, according to the mesh definition criteria presented in Troina et al. (2020). The result can be seen in Fig. 7.

According to Fig. 7, from the $M_4$ mesh with finite element size of 12.5 mm the values converged for the central deflection of the plate $U_z$. So, this is the size of the finite element determined to discretize all other cases (see Fig. 6).

In a first analysis, Figs. 8 to 11, respectively, for P(2,2), P(2,3) P(3,2) and P(3,3), show the results of central and maximum deflection for each spacing situation (see Fig. 6) as a function of the variation in the $h_s/t_s$ ratio.
Figure 8 - Plate P(2,2): (a) Central Deflection; (b) Maximum Deflection

Figure 9 - Plate P(2,3): (a) Central Deflection; (b) Maximum Deflection

Figure 10 - Plate P(3,2): (a) Central Deflection; (b) Maximum Deflection
As already expected, in Figs. 8 to 11 one can note that transforming a portion of material from a non-stiffened plate into stiffeners, keeping the total volume of material constant as recommended by the CDM, a significant reduction of deflections can be achieved. This aspect is proven since all the central and maximum deflection values found for the stiffened plates are less than the values found for the reference plate \( U_z = U_{zMax} = 0.697 \) mm. In addition, it is noticed that the increase in the ratio \( h_s/t_s \) leads to lower deflections. This can be explained due the fact that greater ratios \( h_s/t_s \) imply a greater moment of inertia of the stiffeners cross-section, increasing the plate stiffness and consequently reducing the deflections. These behaviors above mentioned were also observed in Troina et al. (2020).

Still observing Figs. 8 to 11, it can be seen that the variation of transverse \( S_t \) and longitudinal \( S_l \) spacing, as those decrease, there is also a decrease of the central deflections \( U_z \). However, when dealing with maximum deflections, it was possible to notice that the smallest deflections occurred for equidistant spacing for the stiffeners.

For a better understanding of the results presented, a second analysis proposes for both central and maximum deflection, curves with the same \( h_s/t_s \) ratio as function of the spacing variation for each plate arrangements considered in the study, as shown in Figs. 12 to 15.
Figure 12 - P(2,2): (a) $h_s/t_s = 1.06$; (b) $h_s/t_s = 3.37$; (c) $h_s/t_s = 5.26$; (d) $h_s/t_s = 20.84$.

Figure 13 - P(2,3): (a) $h_s/t_s = 1.16$; (b) $h_s/t_s = 2.90$; (c) $h_s/t_s = 4.53$; (d) $h_s/t_s = 17.91$. 
Figure 14 - P(3,2): (a) $h_s/t_s = 1.01$; (b) $h_s/t_s = 2.53$; (c) $h_s/t_s = 3.95$; (d) $h_s/t_s = 27.72$.

Figure 15 - P(3,3): (a) $h_s/t_s = 1.17$; (b) $h_s/t_s = 3.53$; (c) $h_s/t_s = 4.61$; (d) $h_s/t_s = 35.00$. 
The Fig. 12 shows for the P(2,2) arrangement the behavior of central and maximum deflections as a function of the variation of the spacing for ratios $h_s/t_s = 1.06$, $h_s/t_s = 3.37$, $h_s/t_s = 5.26$ and $h_s/t_s = 20.84$. It is possible to observe that the plates with optimized geometry, i.e., with lower deflections, tend to be those that the stiffeners have their position with equal spacing ($S_t = 333.33$ mm and $S_l = 666.66$ mm) except for small $h_s/t_s$ ratios, between 1 and 2. The smallest deflections found occurred for spacing $S_t = 333.33$ mm and $S_l = 666.66$ mm with $h_s/t_s = 20.84$ (see Fig. 12d), which showed $U_z = 0.0275$ mm and $U_{z_{\text{Max}}} = 0.0400$ mm, which represents compared to the reference plate a reduction of 96.05% for central deflection and 94.26% for maximum deflection.

Regarding the arrangement P(2,3), the Fig. 13 shows the curves for the ratios $h_s/t_s = 1.16$, $h_s/t_s = 2.90$, $h_s/t_s = 4.53$ and $h_s/t_s = 17.91$ in relation to the variation of spacing between stiffeners. The smallest deflections occurred for the highest ratio $h_s/t_s = 17.91$, where for the central deflection the lowest value reached was $U_z = 0.0132$ mm for $S_t = 111.11$ mm and $S_l = 666.66$ mm, representing a reduction of 98.10% in relation to the reference plate. In its turn, regarding the maximum deflection, the lowest value was found for $S_t = 333.333$ mm and $S_l = 1000.00$ mm, with $U_{z_{\text{Max}}} = 0.0314$ mm, i.e., a value 95.50% lower than that found for the non-stiffened reference plate.

Observing the results of Fig. 14, with the results for the arrangement P(3,2) taking account the ratios $h_s/t_s = 1.01$, $h_s/t_s = 2.53$, $h_s/t_s = 3.95$ and $h_s/t_s = 27.72$ as function of the variation of the spacing, it is noticed that for ratios with $h_s/t_s$ less than 3 the central and maximum deflection curves practically overlap, showing that the maximum deflections occur in the central region of the plate. The lowest value of central deflection achieved $U_z = 0.0122$ mm for the ratio $h_s/t_s = 27.72$ with $S_t = 333.33$ mm and $S_l = 222.222$; allowing a reduction of 98.25% if compared to the reference plate. For the maximum deflection, the lowest value found was $U_{z_{\text{Max}}} = 0.0199$ mm also for the ratio $h_s/t_s = 27.72$, however for spacing of $S_t = 500.00$ mm and $S_l = 666.67$ mm, representing a minimization of 97.15% in comparison to the reference plate.

Ultimately, the Fig. 15 shows the results of the arrangement P(3,3) considering $h_s/t_s = 1.17$, $h_s/t_s = 3.53$, $h_s/t_s = 4.61$ and $h_s/t_s = 35.00$ in relation to

the stiffeners spacing variation. According to the pattern already observed, the smallest deflections occurred for the highest ratio $h_s/t_s = 35.00$. Regarding the central deflection, the best result found was $U_z = 0.0096$ mm for $S_t = 333.33$ mm and $S_l = 666.67$ mm, reaching a reduction of 98.62%. While for the maximum deflection the lowest value of $U_{zMax} = 0.0161$ mm achieved with $S_t = 500.00$ mm and $S_l = 1000.00$ mm, being 97.70% less than the value found to the reference plate.

4 CONCLUSIONS

Applying the CDM, different arrangements of stiffened plates were proposed, being solved by the application of FEM. After that, through the ES it was evaluated the influence of the variation of the spacing between the stiffeners in terms of central and maximum deflections occurred in the plates.

First of all, it can be concluded that transforming a portion of material deducted entirely from a non-stiffened plate into stiffeners, keeping the total material volume constant, improves considerably the mechanical behavior of the plates, being able to achieve reductions greater than 95% in the central and maximum deflections. Still as expected, the raise in the $h_s/t_s$ ratio caused a significant stiffness improvement in all analyzed plates.

It was also possible to notice that the smallest central deflections were achieved when there was a decrease in the $S_t$ and $S_l$ spacing. Thus, the optimized configuration that best minimized the central deflection was the plate $P(3,3)$ for the ratio $h_s/t_s = 35.00$ for the situation of spacing $S_t = 333.33$ mm and $S_l = 666.67$ mm.

However, concerning the maximum deflection, the lowest values found occurred where the stiffeners were equally spaced in both directions (transverse and horizontal). The best performance achieved in terms of maximum deflection was for plate $P(3,3)$ with ratio $h_s/t_s = 35.00$ and $S_t = 500.00$ mm and $S_l = 1000.00$ mm.

In future works it would be interesting to analyze other volumetric fractions $\phi$, as well as other arrangements with different amounts of stiffeners and spacing.
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